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B. E. (Third Semester) Examination, Nov.-Dec. 2021

(New Scheme)

(IT Branch)

DISCRETE STRUCTURES

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt all questions. Part (a) from each question is compulsory and carry 2 marks. Attempt any ***two*** *parts (b), (c) & (d) with carries 7 marks each.*

Unit-I

1. (a) Write the converse of the following conditional statement. 2

"If 2 + 2 = 4 then blood is green."

(b) Define principal conjunctive normal form. Obtain principal conjunctive normal form of $p \wedge q$ by using truth table. 7

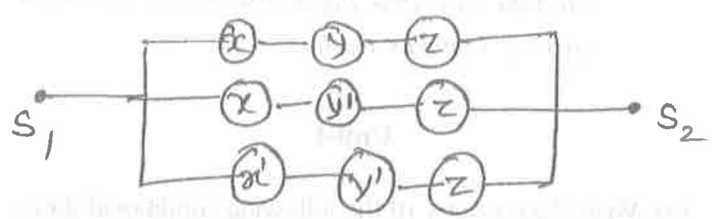
(c) Let $B = \{1, 5, 7, 35\}$ be the set of positive integers and operations '+' and '·' are defined as follows :

$a + b = \text{LCM}(a, b)$

$a \cdot b = \gcd(a, b) \ \forall \ a \in B$

a unary operation '1' on B defined as $a' = 35/a$ $\forall \ a \in B$, show that $(B, +, \cdot, 1)$ is a Boolean algebra. 7

(d) Write the Boolean expression of the following switching circuit in fig. and draw the simplified form of circuit. 7



Unit-II

2. (a) Give an example of a relation which is symmetric and transitive both but not reflexive. 2

(b) Define Cartesian product of two sets. If A, B, C are any three non-empty sets, then

P.T. $A \times (B \cup C) = (A \times B) \cup (A \times C)$ 7

(c) Define composition function. If the function $f : R \to R$ is defined by $f(x) = x^2 - 2x - 3$ and the function $g : R \to R$ is defined by $g(x) = 3x - 4$, then find $g \circ f(x)$ and $f \circ g(x)$. 7

(d) Answer the question for the poset $(\{3, 5, 9, 15, 24, 45\}, a/b)$ with respect to divisibility relation 7

(i) Find the maximal elements.

(ii) Find the minimal elements.

(iii) Is there a greatest and least elements?

(iv) Find the upper bounds $\{3, 5\}$.

(v) Find the least upper bound (sup) $\{3, 5\}$ if they exists.

(vi) Find the lower bound $\{15, 45\}$.

(vii) Find the *glb* (inf) of (15, 45) if they exists.

Unit-III

3. (a) Find the order of every element in the multiplicative group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$. 2

(b) State and prove Lagrange's theorem. 7

(c) If R be the additive group of all real numbers and R_t be the multiplicative group of real numbers then show that the following mappings are isomorphism : 7

$$f : R \to R_t \text{ defined by } f(x) = e^x \ \forall\ x \in R$$

(d) Define the following : 7

(i) Group code

(ii) Ring

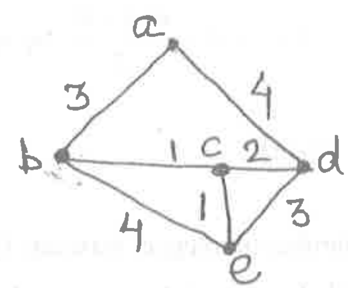
(iii) Integral Domain

Unit-IV

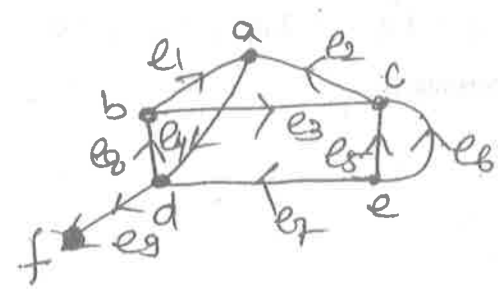
4. (a) Does there exists a 4-regular graph on 6-vertices, if so construct a graph. 2

(b) Show that the maximum number of edges in a simple graph with n-vertices is $\frac{n(n-1)}{2}$. 7

(c) Show that Kruskal's algorithm, find a minimal spanning tree for the graph of fig. 7



(d) Define incidence matrix for diagram and find the incidence matrix of the following diagram. 7



Unit-V

5. (a) Find the number of ways of putting 5 letters in five addressed envelope such that no letter is placed in the right envelope. 2

(b) Define principle of mathematical induction. Show that $1+2+3+...+n=\frac{n(n+1)}{2}$ by mathematical induction $(n \geq 1)$. 7

(c) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. 7

(d) Give that $a_0 = 1$, $a_1 = -2$ and $a_2 = 1$ satisfy the following recurrence relation for $r \geq 3$.

$$a_r + 3\,a_{r-1} + 3\,a_{r-2} + a_{r-3} = 0$$

Determine a_r. 7